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THE UNIVERSITY OF WOLLONGONG

DEPARTMENT OF ECONOMICS

**EXCHANGE RATE VARIABILITY:
A CASE OF NON-LINEAR RATIONAL
EXPECTATIONS?**

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ABSTRACT

A number of authors have recently shown that nominal spot exchange rates closely approximate random walks and that out-of-sample predictions of the random walk model are as good, if not better, than those of most economic models.

This paper develops an alternative economic explanation of exchange rate behaviour. We begin by modifying a simple mean-variance optimising model of Begg (1984) to incorporate rational expectations in both the first and second moments of the distribution of future exchange rates. By taking rational expectations of the conditional variance we obtain a simple nonlinear difference equation which describes the dynamic evolution of the equilibrium exchange rate.

The properties of these rational expectations paths are then considered in detail. It is observed that (similar to many linear rational expectations saddlepoint solutions) convergence and uniqueness are not necessarily guaranteed and so it is important to consider the conditions which are necessary to achieve stability. Finally, it is shown that for particular parameter values, the equilibrium exchange rate may appear to follow seemingly random paths. This phenomenon is not due to any stochastic influence, but is a result of the dynamic non-linear nature of the deterministic model.

1. INTRODUCTION

In this paper we develop an alternative view of exchange rate determination. We do this in response to the following; firstly, Roll (1979), Frenkel (1981), Darby (1983), Adler and Lehman (1983) and others, including the Australian study by Sheen (1986), have found that exchange rates (nominal and real) appear to follow random walks in that they have no systematic components. Indeed many well known studies which support this view also show that economic fundamentals have little effect. For example Meese and Rogoff (1983, p 3) found that a "random walk model would have predicted major country exchange rates....as well as any of our candidate models" in terms of out-of-sample predictions, whilst Adams and Boyer (1986, p 1) claim that exchange rates have followed a random walk and "that the monetary model is an inappropriate specification" due to the lack of any intrinsic dynamics. Finally, Frankel and Foot (1987, p 151) conclude that "it seems likely that the actual spot rate process is more complicated than any of the models tested here".

However, Hakkio (1986) argues that tests of random walks have low power in discriminating between alternatives and therefore are unable to reject the null hypothesis that the exchange rate follows a random walk. Clearly, the matter is not settled and further economic modelling of exchange rates is required.

The second reason for developing an alternative model is to more explicitly consider the stability properties of a multiple equilibrium perfect foresight model of exchange rate determination. The deterministic forward solution of the relevant dynamic price expectations equation is generally not unique in that an infinity of solution paths exist. This is due to the expectations of future endogenous variables influencing current behaviour given rational expectations. Whilst some argue that this non-uniqueness is a fundamental weakness of the rational expectations hypothesis, it is more correct to say it is attributable to the dynamics of models involving expectations and not to the rationality assumption. A popular solution to this indeterminacy is to impose a variety of restrictions, many of which are ad-hoc, to obtain the saddlepath. Another approach considers non-saddlepaths as rational bubbles and whilst this notion has generated recent interest, it has been mostly in empirical testing with little subsequent theoretical developments. In any case Flood and Hodrick (1986), Hamilton (1986) and Meese

(1986) argue that care must be taken in interpreting data which appear to support the existence of speculative bubbles. It appears that apparent bubbles are more likely to be attributed to misspecification of unobserved non-stationarity in market fundamentals. It is clear that further research is required on the convergence of solution paths and how these paths are determined. Burmeister (1980, pp 800-801) claims that "one of the most crucial issues.....concerns the dynamic properties of rational expectations paths and the manner in which the stability properties of these expectations serve to make determinate the stochastic properties of the actual variables".

The purpose of this paper is to further the above discussion by developing a simple non-linear, rational expectations model which describes the dynamic evolution of the equilibrium exchange rate. This allows the analysis of some interesting properties of this alternative model, in particular, the requirements for convergence and uniqueness of the deterministic solution paths. The following section develops the model using mean-variance analysis. It is important to note that it is for illustrative purposes only and a number of equally useful variations could be developed. Section 3 applies the model to Australian/US weekly exchange rate data.

2. AN ALTERNATIVE EXCHANGE RATE MODEL

In order to keep the model simple we modify Begg's (1984) mean-variance model. We adopt this procedure because the important parameters are not determined arbitrarily but are constrained according to optimising behaviour. We define the generalised risk measure (R) such that the approach is equivalent to the expected utility theory. That is:

$$E(U(i)) = U(r) - R \quad (1)$$

where i is total yield and r is its mean. If a Taylor's expansion about i exists for $U(i)$ which is sufficiently large to include all i and if all central moments exist then:

$$R = \sum_{k=2}^{\infty} U^{(k)}(r)/k! - r_k \quad (2)$$

where r_k is the k th central moment of the probability distribution and U^k is the k th derivative of $U(i)$ at r .

For the negative exponential function:

$$U(i) = -\exp(-bi), \quad b > 0 \quad (3)$$

$$\text{such that } R = (b^2\sigma^2/2! - b^3\sigma^3/3! - \dots) \exp(-br) \quad (4)$$

We assume order 3 and above are insignificant. This may not be true, in which case, the mean-variance approach only approximates expected utility.

Another problem is that whilst the chosen utility function reflects risk aversion (b), it restricts absolute risk aversion to be constant, which is unrealistic but keeps matters relatively simple. The preferred alternative is the family of constant elasticity functions, which allow decreasing absolute risk aversion such that changes in wealth (via changes in the balance of payments and government policies) affect behaviour. Unfortunately the model becomes very complex and attempts are at present being made to obtain tractable results. For the purpose of this paper we will contain our attention to the simpler negative exponential utility function.

We assume wealthholders can hold a safe domestic asset, whose one period return is r_t , and a risky foreign denominated asset, whose one period return is r_t^f . The proportion of the total wealth which is held in foreign assets is denominated by F_t . The total proportional yield on the portfolio (i_t) is given by

$$i_t = F_t (s_{t+1} - s_t + r_t^f s_{t+1} + a_t) + (1 - F_t) r_t \quad (5)$$

where s_t is the spot exchange rate (defined as the price of foreign exchange), s_{t+1} the next period spot rate and a_t a premium which reflects risk and liquidity factors, brokerage costs,

withholding tax, etc. Equation (5) includes uncertainty in the form of unknown capital gain or loss on holding foreign assets. We assume that:

$$e_{t+1} \sim N(s_{t+1}^e, \sigma_{e_{t+1}}^2) \quad (6)$$

that is, e_{t+1} is distributed according to its expected value (denoted by superscript e) and its variance, which are both conditional on information at time t .

Wealthholders choose assets so as to maximise expected utility given in equation (1), which is equivalent to choosing F_t to maximise:

$$F_t(s_{t+1}^e - e_t + r_t^f s_{t+1}^e + a_t) + (1 - F_t s_t) r_t - b F_t (1 + r_t^f)^2 \sigma_{s_t}^2 / 2 \quad (7)$$

Differentiating (7) with respect to F_t and setting to zero gives:

$$s_t = D_t(s_{t+1} - b F_t (1 + r_t^f) \sigma_{e_{t+1}}^2 + a_t) \quad (8)$$

where $D_t = (1 + r_t^f) / (1 + r_t)$ represents the ratio of relative discount factors.

In order to obtain the rational expectations solution we follow Begg (1984) and assume the supply of foreign denominated assets and wealth are fixed such that $(F_t = F)$ and D_t has deterministic (d) and stochastic (u_t) components:

$$D_t = d + u_t \text{ where } u_t \sim N(0, \sigma_u^2) \quad (9)$$

Leading equation (8) by one period, taking conditional expectations on s_{t+1} and σ^2 and solving for the general case gives:

$$s_{t+j}^e = a_d d + d s_{t+j+1}^e - J_0(s_{t+j+1}^e)^2 = f(s_{t+j+1}^e) \quad (10)$$

$$J_0 = bF(1 + rf_{t+j+1}) (\sigma_u^2/d^2) \quad (11)$$

which describes the rational expectations forward evolution of equilibrium exchange rates. These non-linear characteristics (which are due to the taking of rational expectations of the second moment of the distribution of future exchange rates) give rise to interesting properties, which we now wish to consider.

Solving equation (10) for the steady state (s^*) gives:

$$s^* = (d + ((D - 1)^2 + 4a_t d^2 J_0)^{1/2}) / 2dJ_0 \quad (12)$$

Clearly the proportionate supply of foreign assets (F), risk aversion (b), risk premium (a) and/or interest rate uncertainty (σ_u^2) will cause an appreciation of the equilibrium expected exchange rate.

The stability or other wise of the steady state can be determined by the derivative of (12) which is:

$$f'(s^*) = 1 - ((d_1 - 1)^2 + 4ad^2 J_0)^{1/2} < 1 \quad (13)$$

The case of $-1 < f'(s^*) < 0$ is shown in Figure 1 and describes a locally unstable steady state. Under these circumstances stability requires the rational expectations saddlepath solution, where the unique rational expectations equilibrium is given by $s_{t+j}^e = s^*$ and the spot rate jumps instantaneously to

$$s_t = s^* + s^* (u_t/d_1) \quad (14)$$

This is necessary because if the forward evolution of expectations moved off the saddlepath then an explosive spiral would result. When $0 < f'(s^*) < 1$ the non-saddlepath describes a monotonic explosive solution and so the saddlepath solution would prevail.

For values of $f'(e^*) < -1$ there is an infinity of possible rational expectations paths. As indicated in Figure 2, any initial belief will generate an indeterminate number of expectation paths. Convergence will not be unique or even guaranteed.

Finally, it can be seen from equation (13) that any increase in the risk premium (a), interest rate uncertainty (σ_n^2) and/or risk aversion (b) will increase the possibility of indeterminacy. We now wish to apply these principles to Australian exchange rate data.

3. AN APPLICATION TO AUSTRALIAN EXCHANGE RATES

3.1 A Useful Modification

We repeat our earlier point that our extension of Begg's (1984) model is for illustrative purposes only and a number of equally useful alternatives could be developed. We do not maintain that the model is the final explanation of exchange rate behaviour, but only that it is a useful device for introducing the concept of non-linear deterministic expectation formation in an optimising framework.

To this end we will find it easier to work with a transformed version of the above model, as detailed by van der Ploeg (1985). Consider

$$Y_{t+1} = Cy_t - Cy_t^2 \quad (15)$$

$$\text{where } y^* = (J_2 s_t^* - J_3)/(1 - 2J_3) \quad (16)$$

$$\text{and } J_3 = ((d - 1) - ((J_d - 1)^2 + 4ad^2J_0)^{1/2})/2d$$

$$C = d_1 (1 - 2J_3) \quad (18)$$

Equation (15) is the transformation of equation (10) according to (16), (17) and (18) and is now in the functional form which has seen relatively new developments in mathematics. Indeed this form of representing dynamic behaviour has received recent theoretical attention and has provided some interesting applications in the biological and physical sciences. More recently Day (1982, 1983), Begg (1984), van der Ploeg (1985) and Melese and Transue (1986) have considered equation (15) in economic contexts.

It can be shown that the derivative of equation (15) $f'(y^*) = f'(s^*) = 2 - C$ where C is in the range $1 < C < 4$ for positive y_t . Equation (15) reverses the direction of the arrows in Figures 1 and 2 and so we are now interested in values of C which cause explosive paths, as in Figure 2. Van der Pleog (1985) has calculated values of C which obtain indeterminate and unique (unstable and stable, respectively) equilibrium paths.

Table 1

Description	Value of C
Unique equilibrium monotonic path	$1 < C < 2$
Unique equilibrium jump path	$1 < C < 3$
Indeterminate path with	
- even period cycles	$3 < C < 3.6786$
- odd period cycles	$3.6786 < C < 3.8284$
- no defined cycles (aperiodic)	$3.8284 < C < 4$

Unique convergent paths exist for C less than 3. For values of C greater than 3 an infinity of possible non-explosive, but non-convergent paths exist. For example when C is in the range $3 < C < 3.4494$, the equilibrium path follows a 2 period limit cycle. This locally unstable, but globally stable, behaviour is shown in Figure 3.

Figures 4 and 5 show aperiodic behaviour for $C = 3.8285$. Note the similarity of the time series path to that of a random walk. This is not due to any stochastic influence or the failure of rational expectations, but is a direct result of the dynamics of the deterministic optimising economic model.

Obviously lower values of C are preferred and since:

$$\begin{aligned}
 C &= 2 - f'(S^*) \\
 &= 1 + (J_1 - 1)^2 + 4ad^2J_0)^{1/2}
 \end{aligned}
 \tag{19}$$

then it can be seen from (11) that lower values of the risk premium (a), interest rate uncertainty, risk aversion and the proportion of wealth held in the risky asset (J_0), will all reduce C .

3.2 Australian Exchange Rates

Weekly observations of the spot \$US/\$A and YEN/\$A exchange rates from 16 May, 1985 to 23 July, 1987 are graphed in Figure 6. We include the YEN/\$A observations to indicate the relative variability of the \$US/\$A spot rates over this period.

Maximum likelihood estimation was used to estimate C in equation (15), for the full period of 113 weekly \$US1 \$A observations. This procedure was repeated for four, somewhat arbitrarily chosen sub-periods and the results are shown in Table 2.

Figure 7 plots the actual and predicted \$US/\$A exchange rates for the first regression over the full period. The mean square error and coefficient of determination values are 0.0675 and 0.9903 respectively.

It can be seen from Table 2 that, with only one exception (10 July, 1986 - 5 February, 1987), all estimates of C are significantly greater than 3 at the 1% level. Those results imply that the rational expectations process follows a locally unstable 2 period limit cycle. We now call this phenomenon a rational speculative bubble. The important difference

Table 2

Period	No. of Observations.	\hat{C}	$S_{\hat{C}}$	t	95% Confidence Interval	MSE
16.5.85-23.7.87	114	3.143	0.029	108.4*	3.09-3.20	0.0675
16.5.85-5.12.85	30	3.204	0.030	106.8*	3.14-3.27	0.0345
5.12.85-10.7.86	30	3.351	0.040	83.8*	3.27-3.43	0.0471
10.7.86-5.2.87	30	2.772	0.025	110.9°	2.72-2.82	0.0322
5.2.87-23.7.87	24	3.363	0.040	84.1*	3.28-3.44	0.0413
* Denotes C is significantly greater than 3 at the %1 level of significance						
° Denotes C is significantly less than 3 at the %1 level of significance						
(SAS was used for all estimation)						

between this type of bubble and other deterministic bubbles is that it is non-explosive. Consequently the criticisms, of bubbles having global instability "built-in", are not appropriate here.

Finally, open market operations in foreign exchange by the authorities will change the proportion of wealth held in foreign assets (F). Any change in F_t will, via changing J_0 in equation (11), cause a change in the steady state exchange rate s^* (equation 12). This can be seen in Figure 3 as a shift in the position of the parabola, giving a new intersection. The value of C will also change (equation (19) and if it is (now) greater than 3 then speculative bubble behaviour will be observed. Accordingly, policy actions may result in rational bubble behaviour. Moreover, any actions which change the risk premium (a), risk aversion (b) and/or the level of interest rate uncertainty (s_u^2), will have feedback effects.

It is now acknowledged that the authorities have been active in the foreign exchange market during 1987, in order to effect the level and variability of exchange rates. Our results for the period 5 February, 1987 to 23 July, 1987 imply that the effects (and therefore the successfulness or otherwise) of this activity are perhaps more complicated than previously believed.

4. CONCLUSION

This paper suggests an alternative view of exchange rate determination. We have done this because existing economic models have at best been only partially successful in predicting exchange rate movements. Our major innovation is the inclusion of non-linear dynamic rational expectation formation, which is not developed in an ad-hoc manner, but is the product of individual optimising behaviour. We do this by modifying models of Begg (1984) and van der Ploeg (1985) which also allow the application of some recent developments in the physical sciences, in order to analyse the dynamic, non-linear properties. In this context we develop the notion of a particular form of deterministic bubble.

The theory is then applied to \$US/\$A weekly exchange rate observations from 5 February, 1987 to 23 July, 1987. The initial results indicate that the spot exchange rate exhibited rational speculative bubble behaviour during this period. Moreover, it is suggested that attempts by the authorities to stabilise rates may lead to reactions which are quite complex in nature.

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